

# Odd Scalar Curvature in Anti-Poisson Geometry

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## Abstract

Recent works have revealed that the recipe for field-antifield quantization of Lagrangian gauge theories can be considerably relaxed when it comes to choosing a path integral measure  $\rho$  if a zero-order term  $\nu_\rho$  is added to the  $\Delta$  operator. The effects of this odd scalar term  $\nu_\rho$  become relevant at two-loop order. We prove that  $\nu_\rho$  is essentially the odd scalar curvature of an arbitrary torsion-free connection that is compatible with both the anti-Poisson structure  $E$  and the density  $\rho$ . This extends a previous result for non-degenerate antisymplectic manifolds to degenerate anti-Poisson manifolds that admit a compatible two-form.

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# 1 Introduction

The main purpose of this Letter is to report on new geometric insights into the field-antifield formalism. In general, the field-antifield formalism [1, 2, 3] is a recipe for constructing Feynman rules for Lagrangian field theories with gauge symmetries. The field-antifield formalism is in principle able to handle the most general gauge algebra, *i.e.* open gauge algebras of reducible type. The input is usually a local relativistic field theory, formulated via a classical action principle in a geometric configuration space. In the field-antifield scheme, the original field variables are extended with various stages of ghosts, antighosts and Lagrange multipliers — all of which are then further extended with corresponding antifields; the gauge symmetries are encoded in a nilpotent Fermionic BRST symmetry [4, 5]; and the original action is deformed into a BRST-invariant master action, whose Hessian has the maximal allowed rank. The full quantum master action

$$W = S + \sum_{n=1}^{\infty} \hbar^n M_n \quad (1.1)$$

is determined recursively order by order in  $\hbar$  from a consistent set of quantum master equations

$$(S, S) = 0, \quad (1.2)$$

$$(M_1, S) = i(\Delta_\rho S), \quad (1.3)$$

$$(M_2, S) = i(\Delta_\rho M_1) + \nu_\rho - \frac{1}{2}(M_1, M_1), \quad (1.4)$$

$$(M_n, S) = i(\Delta_\rho M_{n-1}) - \frac{1}{2} \sum_{r=1}^{n-1} (M_r, M_{n-r}), \quad n \geq 3. \quad (1.5)$$

Here  $(\cdot, \cdot)$  is the antibracket (or anti-Poisson structure),  $\Delta_\rho$  is the odd Laplacian and  $\nu_\rho$  is an odd scalar, which become relevant in perturbation theory at loop order 0, 1, and 2, respectively. It has only recently been realized that the field-antifield formalism can consistently accommodate a non-zero  $\nu_\rho$  term, thereby providing a more flexible framework for field-antifield quantization [6, 7, 8].

The classical master equation (1.2) is a generalization of Zinn-Justin's equation [9], which allows to set up consistent renormalization (if the field theory is renormalizable). If the theory is not anomalous at the one-loop level, there will exist a local solution  $M_1$  to the next equation (1.3), and so forth. Although the field-antifield formalism in its basic form is only a formal scheme — *i.e.* particularly, it assumes that results from finite dimensional analysis are directly applicable to field theory, which has infinitely many degrees of freedom — it has nevertheless been successfully applied to a large variety of physical models. It has mainly been used in a truncated form of the full set of quantum master eqs. (1.2) – (1.5), where all the following quantities

$$(S, S), (\Delta_\rho S), \nu_\rho, M_1, M_2, M_3, \dots, \quad (1.6)$$

are set identically equal to zero. One can for instance mention the AKSZ paradigm [10, 11] as a broad example that uses the truncated field-antifield formalism (1.6) to quantize supersymmetric topological field theories [12, 13, 14, 15]. Currently, very few scientific works describe solutions with non-zero  $M_n$ 's, primarily due to the singular nature of the odd Laplacian  $\Delta_\rho$  in field theory (again because of the infinitely many degrees of freedom). Nevertheless, it should be fruitful to study generic solutions of the full quantum master equation. See the original paper [1] for an interesting solution with  $M_1 \neq 0$ . Finally, it has in many cases been explicitly checked that the field-antifield formalism produces the same result as the Hamiltonian formulation [16, 17, 18]. The formalism has also influenced work in closed string field theory [19] and several branches of mathematics. The geometry behind the field-antifield formalism was further clarified in Ref. [20, 21, 22, 23].

In this Letter we shall only explicitly consider the case of finitely many variables. Our main result concerns the odd scalar  $\nu_\rho$ , which is a certain function of the anti-Poisson structure  $E^{AB}$  and the density  $\rho$ , cf. eq. (6.1) below. It turns out that  $\nu_\rho$  has a geometric interpretation as (minus 1/8 times) the odd scalar curvature  $R$  of any connection  $\nabla$  that satisfies three conditions; namely that  $\nabla$  is 1) anti-Poisson, 2) torsion-free and 3)  $\rho$ -compatible. This is a rather robust conclusion as we shall prove in this Letter that it even holds for degenerate antibrackets. (Degenerate anti-Poisson structures appear naturally from for instance the Dirac antibracket construction for antisymplectic second-class constraints [7, 21, 24, 25].)

## 2 Anti-Poisson structure $E^{AB}$

An *anti-Poisson* structure is by definition a possibly degenerate  $(2,0)$  tensor field  $E^{AB}$  with upper indices that is Grassmann-odd

$$\varepsilon(E^{AB}) = \varepsilon_A + \varepsilon_B + 1 , \quad (2.1)$$

that is skewsymmetric

$$E^{AB} = -(-1)^{(\varepsilon_A+1)(\varepsilon_B+1)} E^{BA} , \quad (2.2)$$

and that satisfies the Jacobi identity

$$\sum_{\text{cycl. } A,B,C} (-1)^{(\varepsilon_A+1)(\varepsilon_C+1)} E^{AD} (\overrightarrow{\partial}_D^\ell E^{BC}) = 0 . \quad (2.3)$$

## 3 Compatible two-form $E_{AB}$

In general, an anti-Poisson manifold could have singular points where the rank of  $E^{AB}$  jumps, and it is necessary to impose a regularity criterion to proceed. We shall here assume that the anti-Poisson structure  $E^{AB}$  admits a compatible two-form field  $E_{AB}$ , *i.e.* that there exists a two-form field  $E_{AB}$  with lower indices that is Grassmann-odd

$$\varepsilon(E_{AB}) = \varepsilon_A + \varepsilon_B + 1 , \quad (3.1)$$

that is skewsymmetric

$$E_{AB} = -(-1)^{\varepsilon_A \varepsilon_B} E_{BA} , \quad (3.2)$$

and that is *compatible* with the anti-Poisson structure in the sense that

$$E^{AB} E_{BC} E^{CD} = E^{AD} , \quad (3.3)$$

$$E_{AB} E^{BC} E_{CD} = E_{AD} . \quad (3.4)$$

This is a relatively mild requirement, which is always automatically satisfied for a Dirac antibracket on antisymplectic manifolds with antisymplectic second-class constraints [7, 21, 24, 25]. Note that the two-form  $E_{AB}$  is neither unique nor necessarily closed. One can define a  $(1,1)$  tensor field as

$$P^A{}_C \equiv E^{AB} E_{BC} , \quad (3.5)$$

or equivalently,

$$P_A{}^C \equiv E_{AB} E^{BC} = (-1)^{\varepsilon_A(\varepsilon_C+1)} P^C{}_A . \quad (3.6)$$

It then follows from either of the compatibility relations (3.3) and (3.4) that  $P^A{}_B$  is an idempotent

$$P^A{}_B P^B{}_C = P^A{}_C . \quad (3.7)$$

## 4 The $\Delta_E$ Operator

An anti-Poisson structure with a compatible two-form field  $E_{AB}$  gives rise to a Grassmann-odd, second-order  $\Delta_E$  operator that takes semidensities to semidensities. It is defined in arbitrary coordinates as [7]

$$\Delta_E \equiv \Delta_1 + \frac{\nu^{(1)}}{8} - \frac{\nu^{(2)}}{8} - \frac{\nu^{(3)}}{24} + \frac{\nu^{(4)}}{24} + \frac{\nu^{(5)}}{12} , \quad (4.1)$$

where  $\Delta_1$  is the odd Laplacian

$$\Delta_\rho \equiv \frac{(-1)^{\varepsilon_A}}{2\rho} \overrightarrow{\partial}_A^\ell \rho E^{AB} \overrightarrow{\partial}_B^\ell , \quad (4.2)$$

with  $\rho = 1$ , and where

$$\nu^{(1)} \equiv (-1)^{\varepsilon_A} (\overrightarrow{\partial}_B^\ell \overrightarrow{\partial}_A^\ell E^{AB}) , \quad (4.3)$$

$$\nu^{(2)} \equiv (-1)^{\varepsilon_A \varepsilon_C} (\overrightarrow{\partial}_D^\ell E^{AB}) E_{BC} (\overrightarrow{\partial}_A^\ell E^{CD}) , \quad (4.4)$$

$$\nu^{(3)} \equiv (-1)^{\varepsilon_B} (\overrightarrow{\partial}_A^\ell E_{BC}) E^{CD} (\overrightarrow{\partial}_D^\ell E^{BA}) , \quad (4.5)$$

$$\nu^{(4)} \equiv (-1)^{\varepsilon_B} (\overrightarrow{\partial}_A^\ell E_{BC}) E^{CD} (\overrightarrow{\partial}_D^\ell E^{BF}) P_F^A , \quad (4.6)$$

$$\begin{aligned} \nu^{(5)} &\equiv (-1)^{\varepsilon_A \varepsilon_C} (\overrightarrow{\partial}_D^\ell E^{AB}) E_{BC} (\overrightarrow{\partial}_A^\ell E^{CF}) P_F^D \\ &= (-1)^{(\varepsilon_A + 1) \varepsilon_B} E^{AD} (\overrightarrow{\partial}_D^\ell E^{BC}) (\overrightarrow{\partial}_C^\ell E_{AF}) P^F_B . \end{aligned} \quad (4.7)$$

It is shown in Ref. [7] that the  $\Delta_E$  operator defined in eq. (4.1) does not depend on the choice of local coordinates, it does not depend on the choice of compatible two-form field  $E_{AB}$ , and it does map semidensities into semidensities. Moreover, the Jacobi identity (2.3) precisely ensures that  $\Delta_E$  is nilpotent

$$\Delta_E^2 = \frac{1}{2} [\Delta_E, \Delta_E] = 0 . \quad (4.8)$$

Earlier works on the  $\Delta_E$  operator include Ref. [6, 25, 26, 27, 28, 29].

## 5 The $\Delta$ Operator

Classically, the field-antifield formalism is governed by the anti-Poisson structure  $E^{AB}$ , or equivalently, the antibracket

$$(f, g) \equiv (f \overleftarrow{\partial}_A) E^{AB} (\overrightarrow{\partial}_B^\ell g) = -(-1)^{(\varepsilon_f + 1)(\varepsilon_g + 1)} (g, f) . \quad (5.1)$$

Quantum mechanically, the field-antifield recipe instructs one to choose an arbitrary path integral measure  $\rho$ , and to use it to build a nilpotent, Grassmann-odd, second-order  $\Delta$  operator that takes scalar functions into scalar functions. It is natural to build the  $\Delta$  operator by conjugating the  $\Delta_E$  operator (4.1) with appropriate square roots of the density  $\rho$  as follows:

$$\Delta \equiv \frac{1}{\sqrt{\rho}} \Delta_E \sqrt{\rho} . \quad (5.2)$$

In this way the  $\Delta$  operator trivially inherits the nilpotency property from the  $\Delta_E$  operator,

$$\Delta^2 = \frac{1}{\sqrt{\rho}} \Delta_E^2 \sqrt{\rho} = 0 . \quad (5.3)$$

In physical applications the nilpotency (5.3) of  $\Delta$  is important for the underlying BRST symmetry of the theory.

## 6 The Odd Scalar $\nu_\rho$

The odd scalar function  $\nu_\rho$  is defined as

$$\nu_\rho \equiv (\Delta 1) = \frac{1}{\sqrt{\rho}}(\Delta_E \sqrt{\rho}) = \nu_\rho^{(0)} + \frac{\nu^{(1)}}{8} - \frac{\nu^{(2)}}{8} - \frac{\nu^{(3)}}{24} + \frac{\nu^{(4)}}{24} + \frac{\nu^{(5)}}{12} , \quad (6.1)$$

where  $\nu^{(1)}, \nu^{(2)}, \nu^{(3)}, \nu^{(4)}, \nu^{(5)}$  are given in eqs. (4.3)–(4.7), and the quantity  $\nu_\rho^{(0)}$  is given as

$$\nu_\rho^{(0)} \equiv \frac{1}{\sqrt{\rho}}(\Delta_1 \sqrt{\rho}) . \quad (6.2)$$

The second-order  $\Delta$  operator (5.2) decomposes as

$$\Delta = \Delta_\rho + \nu_\rho , \quad (6.3)$$

where  $\Delta_\rho$  is the odd Laplacian (4.2). The nilpotency of  $\Delta$  implies that

$$\Delta_\rho^2 = (\nu_\rho, \cdot) , \quad (6.4)$$

$$(\Delta_\rho \nu_\rho) = 0 . \quad (6.5)$$

The possibility of a non-trivial  $\nu_\rho$  has only recently been observed, cf. Ref. [6, 7, 8]. In the past, the odd scalar term  $\nu_\rho$  was not present due to a certain compatibility relation between  $E$  and  $\rho$ , which was unnecessarily imposed, and which (using our new terminology) made  $\nu_\rho$  vanish. In terms of the quantum master equation

$$\Delta e^{\frac{i}{\hbar}W} = 0 , \quad (6.6)$$

the odd scalar  $\nu_\rho$  enters at the two-loop order  $\mathcal{O}(\hbar^2)$

$$\frac{1}{2}(W, W) = i\hbar \Delta_\rho W + \hbar^2 \nu_\rho , \quad (6.7)$$

which in turn leads to the set of eqs. (1.2) – (1.5).

## 7 Connection

In the next two Sections 7 and 8 we will briefly state our sign conventions and definitions for the covariant derivative and the curvature in the presence of Fermionic degrees of freedom. A more complete treatment can be found in Ref. [8, 30]. Other references include Ref. [31]. Our convention for the left covariant derivative  $(\nabla_A X)^B$  of a left vector field  $X^A$  is [30]

$$(\nabla_A X)^B \equiv (\overrightarrow{\partial}_A^\ell X^B) + (-1)^{\varepsilon_X(\varepsilon_B + \varepsilon_C)} \Gamma_A^B{}^C X^C , \quad \varepsilon(X^A) = \varepsilon_X + \varepsilon_A . \quad (7.1)$$

A connection  $\Gamma_A^B{}_C$  is called *anti-Poisson* if it preserves the anti-Poisson structure  $E^{AB}$ , *i.e.*

$$0 = (\nabla_A E)^{BC} \equiv (\overrightarrow{\partial}_A^\ell E^{BC}) + \left( \Gamma_A^B{}_D E^{DC} - (-1)^{(\varepsilon_B + 1)(\varepsilon_C + 1)} (B \leftrightarrow C) \right) . \quad (7.2)$$

It is useful to define a reordered Christoffel symbol  $\Gamma^A{}_{BC}$  as

$$\Gamma^A{}_{BC} \equiv (-1)^{\varepsilon_A \varepsilon_B} \Gamma_B^A{}_C . \quad (7.3)$$

A *torsion-free* connection  $\Gamma^A_{BC}$  has the following symmetry in the lower indices:

$$\Gamma^A_{BC} = -(-1)^{(\varepsilon_B+1)(\varepsilon_C+1)}\Gamma^A_{CB} . \quad (7.4)$$

A connection  $\Gamma^A_{BC}$  is called  $\rho$ -compatible if

$$\Gamma^B_{BA} = (\ln \rho \overleftarrow{\partial}_A^r) . \quad (7.5)$$

There are in principle two definitions for the divergence  $\text{div} X$  of a Bosonic vector field  $X$  with  $\varepsilon_X=0$ . The first divergence definition depends on the density  $\rho$

$$\text{div}_\rho X \equiv \frac{(-1)^{\varepsilon_A}}{\rho} \overrightarrow{\partial}_A^\ell (\rho X^A) , \quad (7.6)$$

while the second definition depends on the connection  $\nabla$

$$\text{div}_\nabla X \equiv \text{str}(\nabla X) \equiv (-1)^{\varepsilon_A} (\nabla_A X)^A = ((-1)^{\varepsilon_A} \overrightarrow{\partial}_A^\ell + \Gamma^B_{BA}) X^A . \quad (7.7)$$

The  $\rho$ -compatibility condition (7.5) precisely ensures that the two definitions (7.6) and (7.7) coincide, and hence that there is a unique notion of volume [32]. We shall only consider torsion-free connections  $\nabla$  that are anti-Poisson and  $\rho$ -compatible, *i.e.* connections that satisfy the above three conditions (7.2), (7.4) and (7.5). Then the odd Laplacian  $\Delta_\rho$  can be written on a manifestly covariant form

$$\Delta_\rho = \frac{(-1)^{\varepsilon_A}}{2} \nabla_A E^{AB} \nabla_B = \frac{(-1)^{\varepsilon_B}}{2} E^{BA} \nabla_A \nabla_B . \quad (7.8)$$

## 8 Curvature

The Riemann curvature tensor is

$$R^A_{BCD} \equiv (-1)^{\varepsilon_A \varepsilon_B} (\overrightarrow{\partial}_B^\ell \Gamma^A_{CD}) + \Gamma^A_{BE} \Gamma^E_{CD} - (-1)^{\varepsilon_B \varepsilon_C} (B \leftrightarrow C) . \quad (8.1)$$

(Note that the ordering of indices on the Riemann curvature tensor is slightly non-standard to minimize appearances of sign factors.) The Ricci tensor is

$$R_{AB} \equiv R^C_{CAB} = \frac{(-1)^{\varepsilon_C}}{\rho} (\overrightarrow{\partial}_C^\ell \rho \Gamma^C_{AB}) - (\overrightarrow{\partial}_A^\ell \ln \rho \overleftarrow{\partial}_B^r) - \Gamma^C_{AD} \Gamma^D_{CB} = -(-1)^{(\varepsilon_A+1)(\varepsilon_B+1)} R_{BA} . \quad (8.2)$$

## 9 Odd Scalar Curvature

The odd scalar curvature  $R$  is defined as the Ricci tensor  $R_{AB}$  contracted with the anti-Poisson tensor  $E^{AB}$ ,

$$R \equiv R_{AB} E^{BA} = E^{AB} R_{BA} , \quad \varepsilon(R) = 1 . \quad (9.1)$$

We now assert that the odd scalar curvature

$$R = -8\nu_\rho \quad (9.2)$$

of an arbitrary connection  $\nabla$  that is anti-Poisson, torsion-free and  $\rho$ -compatible, is equal to (minus eight times) the odd scalar  $\nu_\rho$ . In particular one sees that the odd scalar curvature  $R$  carries no information about the connection  $\nabla$  used, and it depends only on  $E$  and  $\rho$ . Equation (9.2) was proven for the non-degenerated case in Ref. [8]. The degenerated case is proven in Appendix A.

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## A Proof of the Main Eq. (9.2)

Equation (C.9) in Ref. [8] yields that the odd scalar curvature  $R$  can be written as

$$R = -8\nu_\rho^{(0)} - \nu^{(1)} - \frac{1}{2}R_I, \quad (\text{A.1})$$

where  $\nu_\rho^{(0)}$ ,  $\nu^{(1)}$  and  $R_I$  are defined in eqs. (6.2), (4.3) and (A.2), respectively. Since the expression (A.2) below for  $R_I$  only depends on the torsion-free part of the connection, one does in principle not need the torsion-free condition (7.4) from now on. The heart of the proof consists of the following ten “one-line calculations”:

$$R_I \equiv \Gamma^A_{BC}(E^{CB}\overleftarrow{\partial}_A^r) = \Gamma^A_{BC}((E^{CD}E_{DF}E^{FB})\overleftarrow{\partial}_A^r) = 2R_{II} + R_{III}, \quad (\text{A.2})$$

$$R_{II} \equiv \Gamma^A_{BC}P^C_D(E^{DB}\overleftarrow{\partial}_A^r) = -R_{IV} - \nu^{(2)}, \quad (\text{A.3})$$

$$R_{III} \equiv (-1)^{\varepsilon_A(\varepsilon_C+1)}\Gamma_F^A{}_B E^{BC}(\overrightarrow{\partial}_A^\ell E_{CD})E^{DF} = 2R_{III} + R_V, \quad (\text{A.4})$$

$$R_{IV} \equiv \Gamma^A_{BC}E^{CD}(\overrightarrow{\partial}_D^\ell E^{BF})E_{FA} = R_{VI} - R_{IV}, \quad (\text{A.5})$$

$$R_V \equiv (-1)^{\varepsilon_A\varepsilon_C}\Gamma_F^A{}_B P^B_C(\overrightarrow{\partial}_A^\ell E^{CD})P_D^F = R_{VII} - \nu^{(5)}, \quad (\text{A.6})$$

$$R_{VI} \equiv \Gamma^A_{BC}(E^{CB}\overleftarrow{\partial}_D^r)P^D_A = 2R_{VIII} + R_{IX}, \quad (\text{A.7})$$

$$R_{VII} \equiv (-1)^{(\varepsilon_A+1)(\varepsilon_C+1)}E_{AB}\Gamma^B_{CD}E^{DF}(\overrightarrow{\partial}_F^\ell E^{AG})P_G^C = R_{IV} - R_{VIII}, \quad (\text{A.8})$$

$$R_{VIII} \equiv \Gamma^A_{BC}P^C_D(E^{DB}\overleftarrow{\partial}_F^r)P^F_A = -R_{IV} - \nu^{(5)}, \quad (\text{A.9})$$

$$R_{IX} \equiv (-1)^{\varepsilon_A(\varepsilon_C+1)}\Gamma_G^A{}_B E^{BC}P_A^D(\overrightarrow{\partial}_D^\ell E_{CF})E^{FG} = -R_X - \nu^{(4)}, \quad (\text{A.10})$$

$$R_X \equiv (-1)^{\varepsilon_A}\Gamma_F^A{}_B E^{BC}(\overrightarrow{\partial}_C^\ell E_{AD})E^{DF} = -R_{III} - \nu^{(3)}. \quad (\text{A.11})$$

Here we have used the upper compatibility relation (3.3) for the two-form  $E_{AB}$  in the second equality of eqs. (A.2), (A.7), (A.8), (A.9) and (A.10); the lower compatibility relation (3.4) for the two-form  $E_{AB}$  in the second equality of eq. (A.4); the anti-Poisson property (7.2) for the connection  $\nabla$  in the second equality of eqs. (A.3), (A.6), (A.9), (A.10) and (A.11); and the Jacobi identity (2.3) in the second equality of eqs. (A.5) and (A.8). From these ten relations (A.2)–(A.11), the quantity  $R_{III}$  can be determined as follows:

$$\begin{aligned} -R_{III} &= R_V = R_{VII} - \nu^{(5)} = (R_{IV} - R_{VIII}) + (R_{IV} + R_{VIII}) = 2R_{IV} \\ &= R_{VI} = 2R_{VIII} + R_{IX} = -2(R_{IV} + \nu^{(5)}) + (R_{III} + \nu^{(3)} - \nu^{(4)}) \\ &= 2R_{III} + (\nu^{(3)} - \nu^{(4)} - 2\nu^{(5)}), \end{aligned} \quad (\text{A.12})$$

so that

$$R_{III} = \frac{1}{3}(-\nu^{(3)} + \nu^{(4)} + 2\nu^{(5)}). \quad (\text{A.13})$$

Next,  $R_I$  can be expressed in terms of  $R_{III}$ :

$$\frac{1}{2}R_I = R_{II} + \frac{1}{2}R_{III} = -(R_{IV} + \nu^{(2)}) + \frac{1}{2}R_{III} = R_{III} - \nu^{(2)}. \quad (\text{A.14})$$

Inserting eqs. (A.13) and (A.14) into eq. (A.1) yields the main eq. (9.2):

$$R = -8\nu_\rho^{(0)} - \nu^{(1)} - \frac{1}{2}R_I = -8\nu_\rho^{(0)} - \nu^{(1)} + \nu^{(2)} + \frac{1}{3}(\nu^{(3)} - \nu^{(4)} - 2\nu^{(5)}) = -8\nu_\rho. \quad (\text{A.15})$$

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